ANALYSIS AND SYNTHESIS OF DISTURBANCE OBSERVER AS AN ADD-ON ROBUST CONTROLLER

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Overview

- “disturbance observer (DOB)”
  - dates back to the Japanese article (K. Ohnishi, 1987)
  - has been of much interest in control application community, but did not draw much attention in control theory community
  - original idea is intuitively simple, but less analyzed rigorously

- This talk is about
  - developing theoretical framework for DOB approach, and appreciation of it as a robust control tool
  - presentation of robust stability condition and robust nominal performance recovery
  - discussion about various aspects of DOB, and extension to nonlinear systems
Overview

Problem statement

Nominal model plant $P_n$:
\[ \dot{x} = f(x) + \bar{g}(x)u_r \]
\[ y = \bar{h}(x) \]

Real uncertain plant $P$:
\[ \dot{x} = f(x) + g(x)(u + d) \]
\[ y = h(x) \]

Output feedback controller $C$:
\[ \dot{c} = \Gamma(c, y, r) \]
\[ u_r = \gamma(c, y, r) \]

Inner-loop dynamic controller:
\[ u = u_r + \text{“output of an inner-loop controller (DOB)”} \]

The same input-output behavior!
Overview

Problem statement

- not only just the same steady-state behavior, but also the same transient behavior (with the same IC between the nominal and the real)
- important feature required by industry where settling time, overshoot, etc., should be uniform among many product units
- a sharp contrast to other robust or adaptive redesign approach which possibly changes the transient behavior

The same input-output behavior!
Linear DOB: Intuition
$P(s)$: uncertain, but known relative degree $r \geq 1$

$P_n(s)$: nominal model having the same rel. deg $r$

$C(s)$: outer-loop controller for $P_n(s)$

$Q(s)$: stable low-pass filter having rel. deg $= r$
Linear DOB: Intuitive justification

\[ y(s) = \frac{P(s)P_n(s)}{P_n(s) + (P(s) - P_n(s))Q(s)} u_r(s) + \frac{P(s)P_n(s)(1 - Q(s))}{P_n(s) + (P(s) - P_n(s))Q(s)} d(s) \]

\[ Q(j\omega) \approx 1 \quad \Rightarrow \quad y(j\omega) \approx P_n(j\omega)u_r(j\omega) \]
\[ Q(j\omega) \approx 0 \quad \Rightarrow \quad y(j\omega) \approx 0 \]

(\text{where } u_r(j\omega) \approx 0 \approx d(j\omega))
The argument so far does not present real power and limitations.

- robust stability is still of question
- what about transient performance recovery?
- unanswered “observations from practice”:
  - “It does not work for non-minimum phase plant.”
  - “High bandwidth of $Q(s)$ destabilizes the closed-loop.”
  - “It cannot handle large parameter variation.”
  - “Higher order $Q(s)$ leads to instability.”
- can we extend the DOB-based controller for uncertain nonlinear plants?
Our Starting Point

\[ Q(j \omega) \approx 1 \quad \Rightarrow \quad y(j \omega) \approx P_n(j \omega) u_r(j \omega) \]
\[ Q(j \omega) \approx 0 \quad \Rightarrow \quad y(j \omega) \approx 0 \]

(where \( u_r(j \omega) \approx 0 \approx d(j \omega) \))

\[ \therefore \text{ bandwidth of } Q(s) \text{ should be large} \]

\[ Q(s) = \frac{a_0}{(\tau s)^r + a_{r-1}(\tau s)^{r-1} + \cdots + a_1(\tau s) + a_0} \]
\[ \tau > 0 \quad : \text{sufficiently small} \]
Normal form realization of $Q(s)$

\[ Q(s) = \frac{a_0}{(\tau s)^r + a_{r-1}(\tau s)^{r-1} + \cdots + a_1(\tau s) + a_0} \]

\[
\begin{align*}
\dot{q} &= Q(\tau)q + \frac{a_0}{\tau^r}By, \quad y_q = q_1, \\
\dot{p} &= Q(\tau)p + \frac{a_0}{\tau^r}Bu, \quad y_p = p_1,
\end{align*}
\]

Byrnes-Isidori Normal Form

\[
Q(\tau) = \begin{bmatrix}
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
-\frac{a_0}{\tau^r} & -\frac{a_1}{\tau^{r-1}} & \cdots & -\frac{a_{r-1}}{\tau}
\end{bmatrix}, \quad B = \begin{bmatrix} 0 \\
\vdots \\
0 \\
1 \end{bmatrix}
\]
Representation of $P$ (nonlinear plant)

$P : \quad y = x_1$
$\dot{x}_1 = x_2$
$\dot{x}_2 = x_3$
$\vdots$
$\dot{x}_r = f(z, x) + g(z, x)(u + d)$
$\dot{z} = f_0(z, x)$

$f, g, f_0$: unknown
$d$: unknown disturbance

$\dot{x} = Ax + B[f(z, x) + g(z, x)(u + d)]$
$\dot{z} = f_0(z, x)$
$y = Cx$

$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$
$C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$
Representation of nominal plant $P_n$ (with $Q$)

\[ P_n : \quad \bar{y} = \bar{x}_1 \]
\[ \dot{\bar{x}}_1 = \bar{x}_2 \]
\[ \vdots \]
\[ \dot{\bar{x}}_r = \bar{f}(\bar{z}, \bar{x}) + \bar{g}(\bar{z}, \bar{x})u_r = \bar{f}(\bar{z}, \bar{x}) + \bar{g}^* \hat{u}_r \]
\[ \dot{\bar{z}} = \bar{f}_0(\bar{z}, \bar{x}) \]

$P_n^{-1}$: since $\bar{x} = [\bar{y}, \cdots, \bar{y}^{(r-1)}]'$, 
\[ \dot{\bar{z}} = \bar{f}_0(\bar{z}, [\bar{y}, \cdots, \bar{y}^{(r-1)}]') \]
\[ \hat{u}_r = \frac{1}{\bar{g}^*} [\bar{y}^{(r)} - \bar{f}(\bar{z}, [\bar{y}, \cdots, \bar{y}^{(r-1)}]') ] \]

$P_n^{-1}Q(s)$: since $[\bar{y}, \dot{\bar{y}}, \cdots, \bar{y}^{(r-1)}]' = [y_q, \dot{y}_q, \cdots]' = [q_1, q_2, \cdots, q_r]' = q$, 
\[ \dot{\bar{z}} = \bar{f}_0(\bar{z}, q) \]
\[ \dot{q} = Q(\tau)q + \frac{a_0}{\tau^r}By \]
\[ \hat{u}_r = \frac{1}{\bar{g}^*} [\dot{q}_r - \bar{f}(\bar{z}, q)] \]

\[ \hat{u}_r = \frac{\bar{g}(\bar{z}, \bar{x})}{\bar{g}^*} u_r \]

\[ \hat{u}_p = \frac{1}{\bar{g}^*} [\dot{q}_r - \bar{f}(\bar{z}, q)] \]
Summary: Problem Statement

State-space realization of DOB structure

\[
\begin{align*}
\dot{x} &= Ax + B[f(z, x) + g(z, x)(u + d)] \\
\dot{z} &= f_0(z, x) \\
\dot{z}_c &= f_0(z_c, q) \\
\dot{q} &= Q(\tau)q + \frac{a_0}{\tau^r} Bx_1 \\
\dot{p} &= Q(\tau)p + \frac{a_0}{\tau^r} Bu
\end{align*}
\]

\[u = p_1 - \frac{1}{\bar{g}^*} (\dot{q}_r - \bar{f}(z_c, q) - \bar{g}^* \hat{u}_r)\]

\[\hat{u}_r = \frac{\bar{g}(z_c, q)}{\bar{g}^*} u_r\]

\[y = x_1\]

\[u_r \rightarrow P_n \rightarrow y\]

\[r \rightarrow C(s) \rightarrow \hat{y}_r \rightarrow u \rightarrow \hat{u}_r \rightarrow d \rightarrow u_p \rightarrow P(s) \rightarrow y\]

\[? \mid \tau \ll 1\]
*(too brief) Review of Tikhonov’s Theorem*

\[
\begin{align*}
\dot{x} &= f(x, y, u, \varepsilon) \\
\varepsilon \dot{y} &= g(x, y, u, \varepsilon) \\
0 < \varepsilon &\ll 1
\end{align*}
\]

: Standard singularly perturbed form 
\(x: \text{slow}, \ y: \text{fast}\)

Tikhonov’s theorem says ...

**Boundary-Layer Subsystem:**
\[y' = g(x, y, u, 0)\]

If asympt. stable, \(y(t) \to h(x, u)\).

**Quasi-Steady-State Subsystem:**
\[\dot{x} = f(x, h(x, u), u, 0)\]
Closed-loop system in the Standard Singular Perturbation Form

With coordinate change given by

\[ \xi_i = \sum_{j=i}^{r} \frac{a_{j-i}}{a_0} \frac{q_j}{\tau^{r-j}} - \frac{x_i}{\tau^{r-i}}, \quad \eta_i = \tau^{i-1} \left( p_i - \frac{1}{\bar{g}^*} q_r(i) \right) \]

the overall closed-loop system is expressed by

\[
\begin{align*}
\dot{x} &= Ax + B \left[ f + g[\eta_1 + \frac{1}{\bar{g}^*}(\bar{f}(z_c, T(\tau, \xi, x)) + \bar{g}u_r) + d] \right], \\
y &= x_1 \\
\dot{z} &= f_0(z, x) \\
\dot{z}_c &= \bar{f}_0(z_c, T(\tau, \xi, x)) \\
\tau \dot{\xi} &= \begin{bmatrix} -a_{r-1} & 1 & 0 & 1 \\ -a_{r-2} & 0 & 1 \\ \vdots \\ -a_0 & 0 \\ 0 \end{bmatrix} \xi - \tau B \left[ f + g(\eta_1 + \frac{1}{\bar{g}^*}(\bar{f} + \bar{g}u_r) + d) \right] \\
\tau \dot{\eta} &= \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ -a_0 & -a_1 & \cdots & -a_{r-1} \end{bmatrix} \eta + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_0 \end{bmatrix} \left[ \left( 1 - \frac{g}{\bar{g}^*} \right) \left( \eta_1 + \frac{1}{\bar{g}^*}(\bar{f} + \bar{g}u_r) \right) - \frac{1}{\bar{g}^*}(f + gd) \right]
\end{align*}
\]

Robust stability of DOB structure with small $\tau$

= Robust stability of Boundary-Layer subsystem

+ Robust stability of Quasi-Steady-State subsystem

controller C

\[ \dot{c} = \Gamma(c, y, r) \]

\[ u_r = \gamma(c, y, r) \]
Quasi-Steady-State Subsystem

Find eq. pnt. with $\tau = 0 \Rightarrow \xi^* = 0$ and $\eta^*_2 = \cdots = \eta^*_r = 0$

$$\eta^*_1 = \frac{1}{g(z, x)} \left[ \left( 1 - \frac{g}{\bar{g}^*} \right) (\bar{f}(z_c, x) + \bar{g}(z_c, x) u_r) - (f(z, x) + g(z, x) d) \right]$$

QSS subsystem:

- It is the same as the disturbance-free nominal closed-loop!
- z-dynamics becomes unobservable.
  - z-dynamics should be ISS (minimum phase in linear case).
- $\xi^* = 0$ implies $q = x$ (so Q-filter with the plant inverse acts as a state observer!)
Robust stability condition:

The following two are Hurwitz polynomials for all $g(z, x)$:

$s^r + a_{r-1}s^{r-1} + \cdots + a_1s + a_0 = 0$

$s^r + a_{r-1}s^{r-1} + \cdots + a_1s + \frac{g(z, x)}{g^*}a_0 = 0$

- Q-filter should be stable.
- No matter how large the uncertainty $g(z, x)$ is, once bounded, there are $a_i$’s so that two polynomials are Hurwitz!  
  
- Semi-global result for boundedness of $g(z, x)$.
Robust stability of the closed-loop with DOB is guaranteed if, on a compact set of the state-space,

1. zero dynamics of uncertain plant is ISS,
2. outer-loop controller stabilizes the nominal plant,
3. coefficients of Q-filter are chosen for the RS condition,
4. $\tau$ is sufficiently small.
Robust Steady-State Performance

- **Nominal closed-loop**
  
  \[ \dot{\bar{z}} = \bar{f}_0(\bar{z}, \bar{x}) \]  
  \[ \dot{\bar{x}} = A\bar{x} + B[\bar{f}(\bar{z}, \bar{x}) + \bar{g}(\bar{z}, \bar{x})u_r] \]  
  \[ \dot{\bar{c}} = \Gamma(\bar{c}, \bar{x}_1, r) \]  
  \[ u_r = \gamma(\bar{c}, \bar{x}_1, r) \]

  Sol. \( \bar{z}(t), \bar{x}(t), \bar{c}(t) \)
  
  with \( \bar{z}(0) = z_c(0), \bar{x}(0) = x(0), \bar{c}(0) = c(0) \)
  
  where \( z_c(0), x(0), c(0) \in \Omega \)

  “(slow) transient” \( \Rightarrow \) “steady-state”

- **Real closed-loop**
  
  \[ \dot{z} = f_0(z, x) \]  
  \[ \dot{x} = Ax + B[f(z, x) + g(z, x)(u + d)] \]  
  \[ \dot{c} = \Gamma(c, x_1, r) \]  
  \[ u_r = \gamma(c, x_1, r) \]

  Sol. \( z(t), x(t), z_c(t), c(t), q(t), p(t) \)
  
  with IC in a compact set

  “(fast) transient” \( \Rightarrow \) “(slow) transient” \( \Rightarrow \) “steady-state”
Robust (slow) Transient Performance

I. Tikhonov’s theorem guarantees that

\[
\text{for given } \epsilon > 0, \exists \tau^* > 0 \text{ s.t., for all } 0 < \tau < \tau^*, \n\| [z_c(t); x(t); c(t)] - [\bar{z}(t); \bar{x}(t); \bar{c}(t)] \| \leq \epsilon, \forall t \geq 0 \n\]

with \([z_c(0); x(0); c(0)] = [\bar{z}(0); \bar{x}(0); \bar{c}(0)]\),

\[
\text{if } \eta(0) \to \eta^*(0) \text{ and } \xi(0) \to \xi^* \text{ as } \tau \to 0 \n\]

(\eta(0) \text{ and } \xi(0) \text{ depend on } \tau).

II. However, \(\eta(0)\) and \(\xi(0)\) become unbounded as \(\tau \to 0\).
Peaking phenomenon reflected in coordinate change

\[ \xi_i = \sum_{j=i}^r \frac{a_{j-i} \cdot q_j}{a_0 \cdot \tau^{r-j}} - \frac{x_i}{\tau^{r-i}} \Rightarrow \xi_{r-2} = \frac{q_{r-2}}{\tau^2} + \frac{a_1 q_{r-1}}{a_0 \tau} + \frac{a_2 q_r - x_{r-2}}{\tau^2} \]
\[ \xi_{r-1} = \frac{q_{r-1}}{\tau} + \frac{a_1 q_r - x_{r-1}}{a_0 \tau} \]
\[ \xi_r = q_r - x_r \]

\[ \eta_i = \tau^{i-1} \left( p_i - \frac{1}{g^*} q_r(i) \right) \Rightarrow \eta = \begin{bmatrix} p_1 \\ \tau p_2 \\ \vdots \\ \tau^{r-1} p_r \end{bmatrix} + T_a \begin{bmatrix} x_1/\tau^r \\ x_2/\tau^{r-1} \\ \vdots \\ x_r/\tau \end{bmatrix} + T_b \begin{bmatrix} q_1/\tau^r \\ q_2/\tau^{r-1} \\ \vdots \\ q_r/\tau \end{bmatrix} \]

\( x(0), p(0), q(0) \) in a compact set \( \Rightarrow \)

\( \begin{align*}
(a) & \quad |\xi(0)|, |\eta(0)| \to \infty \quad \text{as } \tau \to 0 \\
(b) & \quad |\xi(0)| \leq \frac{K}{\tau^{r-1}}, \quad |\eta(0)| \leq \frac{K}{\tau^r}
\end{align*} \)
Robust (slow) Transient Performance

I. Tikhonov’s theorem guarantees that

for given $\epsilon > 0$, $\exists \tau^* > 0$ s.t., for all $0 < \tau < \tau^*$,

$||[z_c(t); x(t); c(t)] - [\bar{z}(t); \bar{x}(t); \bar{c}(t)]|| \leq \epsilon$, $\forall t \geq 0$

with $[z_c(0); x(0); c(0)] = [\bar{z}(0); \bar{x}(0); \bar{c}(0)]$,

if $\eta(0) \rightarrow \eta^*(0)$ and $\xi(0) \rightarrow \xi^*$ as $\tau \rightarrow 0$

($\eta(0)$ and $\xi(0)$ depend on $\tau$).

II. However, $\eta(0)$ and $\xi(0)$ become unbounded as $\tau \rightarrow 0$.

- “peaking phenomenon” of the state $p(t)$ and $q(t)$

is the cause of the unbounded I.C. $\xi(t)$ and $\eta(0)$.

Lesson: Due to peaking phenomenon, it is not true that conventional DOB recovers (slow) transient performance.
Saturating the Peaking Components

[Esfandiari & Khalil, 1992]

Peaking components that affect slow dynamics:

\[
\begin{align*}
\dot{x} &= Ax + B[f(z, x) + g(z, x)(u + d)] \\
\dot{z} &= f_0(z, x) \\
\dot{z}_c &= \bar{f}_0(z_c, q) \\
\dot{q} &= Q(\tau)q + \frac{a_0}{\tau r} Bx_1 \\
\dot{p} &= Q(\tau)p + \frac{a_0}{\tau r} Bu
\end{align*}
\]

Replace with

\[
\dot{z}_c = \bar{f}_0(z_c, \bar{s}_x(q))
\]

and

\[
u = \bar{s}_{\eta_1} \left( p_1 - \frac{1}{g^*} \dot{q}_r \right) + \frac{1}{g^*} \bar{s}_{\bar{x}_r} \left( \bar{f}(z_c, q) + \bar{g}(z_c, q) u_r \right)
\]
Stability Loss in BL Subsystem and its recovery by a dead-zone function

- BL $\eta$-dynamics becomes

$$\tau \dot{\eta} = \begin{bmatrix} 1 & 1 \\ 0 & -a_1 & \cdots & -a_{r-1} \end{bmatrix} \eta + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ a_0 \end{bmatrix} \left( -\frac{g}{g^*} \bar{s}_{\eta_1}(\eta_1) + \ldots \right)$$

and loses GES.

- Add a dead-zone function: $\bar{d}(\eta_1) := \eta_1 - \bar{s}_{\eta_1}(\eta_1)$

  to recover GES.
Stability Analysis of BL Subsystem with saturation & deadzone

\[ \xi' = \begin{bmatrix} -a_{r-1} & 1 & \cdots & 1 \\ -a_{r-2} & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ -a_0 & 0 & \cdots & 0 \end{bmatrix} \xi - \tau B \left[ f + g(\eta_1 + \frac{1}{\bar{g}^*}(\bar{f} + \bar{g}u_r) + d) \right] \]

\[ \eta' = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix} \eta + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_0 \end{bmatrix} \left[ -\frac{g}{\bar{g}^*} s_{\eta_1}(\eta_1) - \bar{d}(\eta_1) + F(\tau, \xi, t) \right] \]

\[ \tilde{\eta} := \eta - \eta^*(t), \quad V_\xi = \xi^T P_\xi \xi \quad V_\eta = \tilde{\eta}^T P_\eta \tilde{\eta} \quad \text{Circle criterion guarantees GES with QLF.} \]

\[ \frac{d}{dt} \left( \alpha V_\xi + V_\eta \right) \leq -\alpha c_1 |\xi|^2 + \alpha \tau c_2 |\xi| - c_3 |\tilde{\eta}|^2 + |\tilde{\eta}| (c_4 \tau + c_5 |\xi|) \]

\[ \left\| \begin{bmatrix} \xi(T) \\ \tilde{\eta}(T) \end{bmatrix} \right\| \leq ke^{-\frac{\lambda T}{\tau}} \left\| \begin{bmatrix} \xi(0) \\ \tilde{\eta}(0) \end{bmatrix} \right\| \leq ke^{-\frac{\lambda T}{\tau}} \left( \frac{K}{\tau} + \tilde{k} \right) + \delta(\tau), \quad T > 0 \]
Application of Tikhonov’s Theorem for the second interval

Let \( X(t) = [z_c(t); x(t); c(t)] \) and \( \bar{X}(t) = [\bar{z}(t); \bar{x}(t); \bar{c}(t)] \).

- \( \epsilon_1 \to 0 \) as \( T \to 0 \)
- \( \epsilon_3 \to 0 \) as \( \epsilon_2 \to 0 \) and \( \tau \to 0 \)
- \( \epsilon_2 = \| X(T) - \bar{X}(T) \| \leq k e^{-\lambda \frac{T}{\tau}} \left( \frac{K}{\tau^r} + \hat{k} \right) + \delta(\tau) \)

\( \epsilon_2 = \| X(T) - \bar{X}(T) \| \to 0 \) as \( \tau \to 0 \)!
Summary:
Robust Transient Performance Recovery

For given $\epsilon > 0$, $\exists \tau^* > 0$ s.t., for all $0 < \tau < \tau^*$,

$$\| [z_c(t); x(t); c(t)] - [\bar{z}(t); \bar{x}(t); \bar{c}(t)] \| \leq \epsilon, \quad \forall t \geq 0$$

with $[z_c(0); x(0); c(0)] = [\bar{z}(0); \bar{x}(0); \bar{c}(0)]$,

thanks to saturation & dead-zone functions.
Proposed Nonlinear DOB Structure

\[ \dot{z}_c = \tilde{f}_0(z_c, \bar{s}_x(q)) \]

\[ \dot{q} = Q(\tau)q + \frac{a_0}{\tau_r} By \]

\[ \dot{p} = Q(\tau)p \]

\[ + \frac{a_0}{\tau_r} B \left( \bar{s}_{\eta 1} \left( p_1 - \frac{1}{g^*} \dot{q}_r \right) + \left( 1 - \frac{1}{g^*} \right) d \left( p_1 - \frac{1}{g^*} \dot{q}_r \right) + \frac{1}{g^*} \bar{s}_{\dot{x}_r} \left( \bar{f}(z_c, q) + \bar{g}(z_c, q) u_r \right) \right) \]

\[ u = \bar{s}_{\eta 1} \left( p_1 - \frac{1}{g^*} \dot{q}_r \right) + \frac{1}{g^*} \bar{s}_{\dot{x}_r} \left( \bar{f}(z_c, q) + \bar{g}(z_c, q) u_r \right) \]
Example: Point Mass Satellite

\[ \dot{\rho} = v \]

\[ \dot{v} = \rho \omega^2 - \frac{K}{\rho^2} + \frac{1}{m}(u_\rho + d_\rho) \]

\[ \dot{\phi} = \omega \]

\[ \dot{\psi} = -\frac{2\nu \omega}{\rho} + \frac{1}{m \rho}(u_\psi + d_\psi), \]

\[
\begin{bmatrix}
    u_\rho \\
    u_\psi
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \cos \theta(t) & -\sin \theta(t) \\
    \sin \theta(t) & \cos \theta(t)
\end{bmatrix}
\begin{bmatrix}
    u_1^+ \\
    u_2^+
\end{bmatrix}
\]

\[ =: J(\theta(t)) u^+ \]

Unknown: \( m, K, d_\rho, d_\psi, \) and \( \theta(t) = \theta_0(t) + \tilde{\theta}(t) \) (\(|\tilde{\theta}(t)| < \pi/4\))

Measurement: \( \rho, \psi, \) and \( \theta_0(t) \)

Goal: \( x_{11} := \rho(t) - \rho_* \) and \( x_{21} := \rho_*(\psi(t) - \omega_* t) \) go to zero

Controller: \([\text{linear state feedback} + \text{linear observer (by linearization)}] + \text{DOB with linear nominal model}\)
Example: Point Mass Satellite

Nominal:

Real: with DOB (solid=nominal)

Real: w/o DOB:
Robust Stability Condition for Linear DOB

- back to linear
- presents simple linear robust stability condition working on frequency domain
Problem Formulation

- **Standing Assumption:** $P(s) \in \mathcal{P}$
  
  where $\mathcal{P}$ is a collection of

  $$
  P(s) = \frac{\beta_{n-r} s^{n-r} + \beta_{n-r-1} s^{n-r-1} + \cdots + \beta_0}{\alpha_n s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_0}
  $$

  $\alpha_i, \beta_i$: bounded variation, $\alpha_n, \beta_{n-r}$: does not change sign

- **Ass:** $C(s)$ is designed for $P_n(s)$.

- **Problem:** Design of $Q(s)$ for robust stability
Characteristic equation with $Q(s)$

We should choose $a_i$’s in

$$Q(s) = \frac{(\tau s)^l + b_{l-1}(\tau s)^{l-1} + \cdots + b_1(\tau s) + a_0}{(\tau s)^{l+r} + a_{l+r-1}(\tau s)^{l+r-1} + \cdots + a_1(\tau s) + a_0} =: \frac{N_Q(s, \tau)}{D_Q(s, \tau)}, \quad \tau > 0$$

so that the nine transfer functions from $[r, d, n]$ to $[\bar{e}, u, \bar{u}]$:

$$\frac{1}{\Delta(s)} \left[ Q(P - P_n) + P_n, (Q - 1)PP_n, (Q - 1)P_n \right] \begin{array}{ccc}
Q(P - P_n) + P_n, & (Q - 1)PP_n, & (Q - 1)P_n \\
CP_n, & (1 - Q)P_n, & -Q - CP_n \\
CPP_n, & (1 - Q)PP_n, & (1 - Q)P_n
\end{array}$$

where $\Delta(s) = (1 + PC)P_n + Q(P - P_n)$

is Hurwitz. Equivalently, with

$$P(s) = \frac{N(s)}{D(s)}, P_n(s) = \frac{N_n(s)}{D_n(s)}, C(s) = \frac{N_c(s)}{D_c(s)}$$

the following Characteristic equation is Hurwitz:

$$\delta(s; \tau) := (DD_c + NN_c)N_nD_Q + N_QD_c(ND_n - N_nD)$$
Robust Stability of the Overall System

Lemma (from Rouche’s theorem): Relation between the roots of 
\[ p(s) = 0 \text{ and } p(s) + \tau q_1(s) + \tau^2 q_2(s) + \cdots + \tau^k q_k(s) = 0. \]

Observation 1: 
\[ \delta(s; \tau) := (DD_c + NN_c)N_nD_Q + N_QD_c(ND_n - N_nD) \]
\[ \delta(s; 0) = a_0 N(s)(D_cD_n(s) + N_cN_n(s)) =: p_s(s) \]

Observation 2: 
\[ \tilde{\delta}(s; \tau) := \tau^m \delta(s/\tau; \tau), \quad m := \deg(DD_cN_n) = \deg(p_s(s)) \]
\[ \tilde{\delta}(s; 0) = cs^m \left[ D_Q(s, 1) + \left( \lim_{s \to \infty} \frac{P(s)}{P_n(s)} - 1 \right) N_Q(s, 1) \right] =: p_f(s) \]

Lemma: As \( \tau \to 0 \),
m roots of \( \delta(s; \tau) \) converge to m roots of \( p_s(s) \),
r roots of \( \delta(s; \tau) \) converge to \( (1/\tau) \) times r roots of \( p_f(s) \).

Theorem: If (and only if)
(1) \( P_nC/(1 + P_nC) \) is stable
(2) \( P(s) \) is of minimum phase for all \( P(s) \in \mathcal{P} \)
(3) \( p_f(s) \) is Hurwitz for all \( P(s) \in \mathcal{P} \)
then \( \exists \tau^* \) so that the overall system is stable for all \( 0 < \tau < \tau^* \).
Design of $Q(s)$: when

$$Q(s) = \frac{a_0}{(\tau s)^r + a_{r-1}(\tau s)^{r-1} + \cdots + a_0}$$

$p_f(s) = D_Q(s; 1) + \left( \lim_{s \to \infty} \frac{P(s)}{P_n(s)} - 1 \right) N_Q(s; 1)$

$$= s^r + a_{r-1}s^{r-1} + \cdots + a_1s + \left( \lim_{s \to \infty} \frac{P(s)}{P_n(s)} \right) a_0$$

cf. robust stability condition for nonlinear case was:

$$s^r + a_{r-1}s^{r-1} + \cdots + a_1s + \frac{g(z, x)}{g^*} a_0 : \text{Hurwitz}$$

How to choose $Q(s)$:

Choose $a_i$’s so that $s^{r-1} + a_{r-1}s^{r-2} + \cdots + a_1 = 0$ is Hurwitz.

Since $0 < \left( \lim_{s \to \infty} \frac{P(s)}{P_n(s)} \right) < \lambda$ with some constant $\lambda$, we can choose sufficiently small $a_0$ so that

$s^r + a_{r-1}s^{r-1} + \cdots + a_1s + \lambda a_0 = 0$ is Hurwitz.
Discussion: Answers to the raised questions

1. It does not work for non-minimum phase plant.
   ✓ Correct, for small $\tau$.

2. Small $\tau$ destabilizes the closed-loop.
   ✓ No, if $Q(s)$ satisfies RS condition. It may be the effect of ‘initial peakings’ but not a destabilization.

3. It cannot handle large parameter variation.
   ✓ No in general. By following the design guidelines, any bounded variation can be handled.

4. Higher order $Q(s)$ leads to instability.
   ✓ No in general. But, for higher order $Q(s)$, the selection of coefficients becomes more complicated.
Experiments

robust stability condition in action

Taken from
• Yi, Chang, & Shen, IEEE/ASME Trans. Mechatronics, 2009
Disturbance-Observer-Based Hysteresis Compensation for Piezoelectric Actuators

Jingang Yi, Senior Member, IEEE, Steven Chang, and Yantao Shen, Member, IEEE

Fig. 4. PMN-PT/PDMS cantilever actuator.
Step responses
Robust tracking control

Hyundai HS-165 (handling 165kg)

\[ \dot{p}_1 = p_2 \]
\[ \dot{p}_2 = -\frac{a_0}{\tau} p_1 - \frac{a_1}{\tau} p_2 + \frac{a_2}{\tau^2} u \]

\[ \dot{q}_1 = q_2 \]
\[ \dot{q}_2 = -\frac{a_0}{\tau} q_1 - \frac{a_1}{\tau} q_2 + \frac{a_2}{\tau^2} \theta_m \]

Robust tracking control

Experiment results

![Graphs showing x position over time for conventional and proposed methods.](image-url)
Summary

- theoretical study on DOB
- robust stability condition
  (new even for classical linear DOB)
- transient performance recovery
  (thanks to saturation/dead-zone functions)
- DOB = Inner-loop controller = Your first AID kit!
THANK YOU
감사합니다

Any feedback or comments are welcome
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